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CALCULUS.

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279. Proposed by L. H. McDONALD, M. A., Ph. D., Sometime Tutor at Cambridge, Jersey City, N. J.

Find the ellipse of minimum area which will pass through the vertices of a triangle. (Hedrick-Goursat's *Math. Anal.*, p. 133, ex. 9.)

I. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

If B be the origin; BA, BC the axes; then from Vol. X, Nos. 8-9, of the MONTHLY, page 204,

$$clx^2+any^2+(al+cn-bm)xy-aclx-acny=0,$$

is the equation to the circum-ellipse, and

$$\frac{2\pi a^2bc^2lmn\sin B}{[4acln-(al+cn-bm)^2]^{\frac{3}{2}}}=\text{its area.}$$

Let $m=ul$, $n=vl$. $\therefore cx^2+avy^2+(a+cv-bu)xy-acx-acvy=0\ldots(1)$.

$$\frac{2\pi a^2bc^2uv\sin B}{[4acv-(a+cv-bu)^2]^{\frac{3}{2}}}=m=\text{minimum.}$$

Now $dm/du=0$ and $dm/dv=0$.

$$\therefore 4acv-(a+cv-bu)^2=3bvu(a+cv-bu),\ \ldots(2).$$
$$4acv-(a+cv-bu)^2=3cvu(a-cv+bu).\ \ldots(3).$$

Hence, $b(b+c)u=c(b+c)v+a(b-c)$, or $u=\dfrac{cv}{b}+\dfrac{a(b-c)}{b(b+c)}\ldots(4)$.

(4) in (2) gives, $3cv^2(b+c)-v[2(b+c)^2-3a(b-c)]+2ac=0$.

$$\therefore v=\frac{2(b+c)^2-3a(b-c)-\sqrt{[2(b+c)^2-3a(b-c)]^2-24ac^2(b+c)}}{6c(b+c)}.$$

$$u=\frac{2(b+c)^2+3a(b-c)-\sqrt{[2(b+c)^2-3a(b-c)]^2-24ac^2(b+c)}}{6c(b+c)}.$$

These values of u and v in (1) give the required ellipse.

Corollary. If $a=b=c$, $u=v=\frac{1}{3}$.

II. **Solution by J. SCHEFFER, A. M.. Hagerstown, Md.**

Let ACB be the triangle; choose $BC=a$ for the axis of abscisses, and CA for that of ordinates. Any circumscribed ellipse is of the form $y^2+Axy+Bx^2+Cy+Dx=0$; and since $(a, 0)$ and $(0, b)$ are points of the ellipse, we have $C=-b$, $D=-Ba$, and the above equation reduces to $y^2+Axy+Bx^2-by-Bax=0$.

Transforming it to the center of the ellipse as origin, it reduces to

$$y^2+Axy+Bx^2-\frac{Bb^2-ABab+B^2a}{4B-A^2}=0.$$

The area is$=\pi\sin C.\ \dfrac{Bb^2-ABab+B^2a}{4B-A^2}=m$. Developing $\dfrac{\partial m}{\partial A}=0$, and $\dfrac{\partial m}{\partial B}=0$, we find $A=\dfrac{b}{a}$ and $B=\dfrac{b^2}{a^2}$, and thus find the ellipse of minimum area to be, $a^2y^2+abxy+b^2x^2-a^2by-ab^2x=0$. The center is the point $(a/3, b/3)$. The maximum ellipse about the triangle is concentric, and its equation is

$$a^2y^2+abxy+b^2x^2-a^2by-ab^2x+\frac{a^2b^2}{4}=0.$$

Also solved by C. N. Schmall, and V. M. Spunar.

280. **Proposed by C. N. SCHMALL, 89 Columbia Street, New York.**

Find the envelope of the system of spheres

$$\left.\begin{array}{r}(x-a)^2+(y-b)^2+z^2=r^2\\ a^2+b^2=c^2\end{array}\right\}.$$

Solution by J. SCHEFFER, A. M., Hagerstown, Md.; G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.; V. M. SPUNAR, Pittsburg, Pa., and the PROPOSER.

Differentiating $(x-a)^2+(y-b)^2+z^2=r^2$ and $a^2+b^2=c^2$ with reference to a as the independent variable, we have $(x-a)+(y-b)\dfrac{\partial b}{\partial a}=0$, and $a+b\,\dfrac{\partial b}{\partial a}=0$; from the second equation we get $\dfrac{\partial b}{\partial a}=-\dfrac{a}{b}$, and substituting in the first we get $(x-a)-\dfrac{a}{b}(y-b)=0$, whence $b=\dfrac{ay}{x}$ and combining this with $a^2+b^2=c^2$, we get $a^2=\dfrac{c^2x^2}{x^2+y^2}$, $b^2=\dfrac{c^2y^2}{x^2+y^2}$, and substituting this in the first given equation, we have, after some easy reductions:

$$x^2+y^2+z^2-2c\sqrt{(x^2+y^2)}=r^2-c^2.$$

This is, as can easily be seen, the equation of the surface of a ring, the central line of which is a circumference whose radius is$=c$, and the perpendicular section of a circle whose radius is$=\sqrt{(2c^2-r^2)}$.